International Journal on Recent Researches in Science, Engineering & Technology (IJRRSET)
A Journal Established in early 2000 as National journal and upgraded to International journal in 2013 and is in existence for the last 10 years. It is run by Retired Professors from NIT, Trichy. Journal Indexed in JIR, DIIF and SJIF.
Available online at: www.jrrset.com
Received on: 04.12.2018 Published on: 20.01.2019 Pages : 15-18
ISSN (Print) : 2347-6729
ISSN (Online) : 2348-3105
Volume 7, Issue 1
January 2019.
JIR IF : 2.54
SJIF IF : 4.334
Cosmos: 5.395

STATE SPACE MODELS AND STRUCTURAL PROPERTIES OF WHEELED ROBOTS

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ABSTRACT

In this section, the mobility analysis discussed in the previous section is reformulated into a state-space form that will be useful for subsequent developments. We introduce four different kinds of state-space representation that are of interest for understanding the behavior of wheeled robots, and for control design purpose. The posture kinematic model, which is the simplest state-space model able to give a global description of the robot, from the users viewpoint. This paper also expresses the structural properties of the above models of wheeled robots from a control design viewpoint. Since, in most situations, the user is only interested in the posture of the robot, and not in the internal variables (such as the wheel orientation angles), the most interesting models are the posture models (kinematic or dynamic). This is why the discussion on structural properties will be mainly based on the posture models.

POSTURE KINEMATIC MODELS

We have shown that, whatever the type of robot, the velocity vector $\dot{\xi}(t)$ is restricted to belong to a distribution Δc defined as

$$\xi \in \Delta c = \mathrm{span}\{\mathrm{col}[R\_(\theta)\Sigma(\beta s)]\}$$

where the columns of the matrix $\Sigma(\beta s)$ constitute a basis of $N[C*1\ (\beta s)]$. This is equivalent to the following statement: for all t, there exists a vector η such that

$$\xi = R^T(\theta)\Sigma(\beta s)\eta$$

The dimension of the distribution Δc, and hence of the vector $\eta(t)$, is equal to the degree of mobility δm of the robot. Obviously, in the case where the robot has no steering wheels, the matrix Σ is constant, and the expression reduces to

$$\xi = R\_(\theta)\Sigma\eta\ .$$

In the opposite case ($\delta s \geq 1$), the matrix Σ explicitly depends on the orientation angles βs, and the expression can be augmented as follows:

$$\xi = R\_(\theta)\Sigma(\beta s)\eta$$
$$\dot{\beta} s = \varsigma$$

The representation can be viewed as a state-space representation of the model, reflecting the mobility restriction induced by the constraints; it is termed the posture kinematic model. The state vector is constituted by the three posture coordinates $\dot{\xi}(t)$ and, possibly, by δs orientation coordinates βs. The vectors **η** and **ξ**, of dimension δm and δs, respectively, are homogeneous to velocities and can be interpreted as control inputs entering the model linearly. Nevertheless, this interpretation should be treated with some care, since the true physical inputs are the torques provided by the embarked actuators.

This posture kinematic model allows us to discuss further the maneuverability of wheeled robots. The degree of mobility δm is equal to the number of degrees of freedom that can be directly manipulated from the inputs **η**(t), without reorientation of the steering wheels. Intuitively, it corresponds to how many degrees of freedom the robot could have instantaneously from its current position, without steering any of its wheels. This number δm is not equal to the overall number of degrees of freedom of the robot that can be manipulated from the inputs **η**(t) and **ς**(t), which is equal to the sum δM = δm+δs andwhich we could call degree of maneuverability. It includes the δs degrees of freedom that are accessible from the inputs **ς**(t). However, the action of **ς**(t) on the posture coordinates $\dot{\xi}(t)$ is indirect, since it is achieved only through the coordinates βs, which are related to the inputs **ς**(t) by an integral action, reflecting the fact that the modification of the orientation of a steering wheel cannot be achieved instantaneously.

The maneuverability of a wheeled robot depends not only on δM, but also on the way these δM degrees of freedom are partitioned into δm and δs. Therefore, two indices are needed to characterize the maneuverability. Obviously the ideal situation is that of omnimobile robots where δM = δm = 3. In order to avoid useless notational complications, we will assume from now on that the degree of steerability is equal to the number of steering wheels, i. e., Ns = δs.

This is a restriction from a robot design viewpoint. However, for the mathematical analysis of the behavior of mobile robots, there is no loss of generality in this assumption, although it considerably simplifies the technical derivation. Indeed, for robots with an excess of steering wheels, it is always possible to reduce the condition to a minimal subset of exactly δs independent constraints that correspond to the δs wheels that have been selected as the master steering wheels and to ignore the other slave wheels in the analysis.

Irreducibility, Controllability, and Nonholonomy

1. We first address the question of the reducibility of the kinematic posture state-space model. A state model is reducible if there exists a change of coordinates such that some of the new coordinates are identically zero along the motion system. For a nonlinear dynamical system without drift like reducibility is related to the dimension of the involutive closure .Δ of the following distribution Δ, expressed in local coordinates as

$$\boldsymbol{\Delta(z) = \text{span}\ [\text{col}B(z)]\ .}$$

A well-known consequence of the Frobenius theorem is that the system is reducible only if **dim(. Δ) ≤ dim(Δ)−1.**

The following property can be checked for the posture kinematic models of wheeled robots. For the posture kinematic model $\dot{\mathbf{z}} = \mathbf{B(z)u}$,

– the input matrix B(z) has full rank, i. e., **rank [B(z)] = δm+δs∀z,**

– the involutive distribution .Δ(z) has constant maximal dimension, i. e., **dim [Δ(z)]= 3+δs.**

As a consequence, the posture kinematic model of a wheeled robot is irreducible. This is a coordinate free property.

This property has another consequence related to the controllability of the posture kinematic model. For a nonlinear dynamical model without drift of the form, the strong accessibility algebra coincides with the involutive distribution $.\Delta(z)$, which has constant maximal dimension. It follows that the strong accessibility rank condition is satisfied and, therefore, the system is strongly accessible from any configuration. For such a driftless system this implies controllability. Practically, this means that a mobile robot can always be driven from any initial posture $\xi 0$ to any final one ξf , in a finite time, by manipulating the velocity control inputs $u = (\boldsymbol{\eta}^T \boldsymbol{\varsigma}^T)$. Finally, the difference between the dimensions of the two distributions $\Delta(z)$ and $.\Delta(z)$, i. e.,

$$\mathbf{Dim}[\Delta(s)] - \mathbf{dim}\ [\Delta(s)] = (3+\delta s)-(\delta m+\delta s)$$
$$= 3-\delta m$$

is related to the nonholonomy of the posture kinematic model. If this difference is nonzero (i. e., if $\delta m \leq 2$) the posture kinematic model is said to be nonholonomic. If $\delta m = 3$, which is the case only for omnimobile robots, the kinematic posture model is holonomic.

2. The configuration kinematic model is obtained from the posture model by adding the evolution of the internal variables $\beta c(t)$ and $\phi(t)$, and takes the same form $q^{\cdot} = S(q)u$. In order to analyze reducibility and controllability issues we now have to consider the following two distributions: $\Delta 1(q) = \text{span}\ [\text{col}(S(q))]$, and its involutive closure $.\Delta 1(q)$. It follows immediately that

$$\delta m + Ns = \dim\ [\Delta 1(q)] \leq \dim[.\Delta 1(q)] \leq \dim(q)$$
$$= 3 + N + Nc + Ns\ .$$

We define the degree of nonholonomy of the configuration kinematic model as

$$M = \dim[.\Delta 1(q)]-(\delta m+\delta s)$$

This number represents the number of velocity constraints that are not integrable and therefore cannot be eliminated from the configuration evolution description, whatever the choice of the generalized coordinates. It must be pointed out that this number depends on the particular structure of the robot, and thus it has not necessarily the same value for two robots belonging to the same class. On the other hand, for a particular choice of generalized coordinates, the number of coordinates that can be eliminated by integration of the constraints is equal to the difference between $\dim(q)$ and $\dim(\Delta 1(q))$.

It can be checked that the configuration kinematic model of all types of wheeled robots (including omnimobile robots) is nonholonomic (i. e., the degree of nonholonomy is not equal to zero), but is reducible. Moreover, it does not satisfy the strong accessibility rank condition. This property does not contradict the irreducibility of the posture kinematic model. The reducibility of the configuration model means that there exists at least one smooth function of $\mathbf{q}(t)$, involving explicitly at least one of the variables $\beta c(t)$ and $\phi(t)$, that is constant along the trajectories of the system compatible with the full set of kinematic constraints.

3. The posture models are related to the corresponding kinematic models, with the difference that the variables are part of the state vector. This implies the existence of a drift term and the fact that the input vector fields are constant. The dynamic models inherit the structural properties of the corresponding dynamic model. In particular, the posture dynamic model is irreducible and small-time locally controllable.

CONCLUSION

The discussion of mobility and the derivation of the models are based on assumptions concerning the contact between the ground and the wheels: it is assumed that pure rolling and nonslip conditions are satisfied for each wheel. These conditions lead to the kinematic constraints that constitute the basis of the analysis, and particularly of the properties related to the non-holonomy of these models. All model-based control designs therefore also rely on the same assumptions. These assumption are an idealization of the physical reality: these kinematic constraints are not satisfied exactly, and the contact effects are characterized by local slipping effects that are related through phenomenological laws to the contact forces.

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